Inertial Navigation Performance Improvement Using Gravity Gradient Matching Techniques

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Theme

RECENTLY there has been considerable interest in the possibility of correcting high performance inertial navigation systems for gravitational anomaly induced errors using gravity gradient measurements aboard the moving vehicle. A combined gravity gradiometer-inertial navigation system integrated for correction of the gravity anomaly will also measure and hence can be used to map the gravitational anomaly field on a real time basis. The primary purpose of this paper is to evaluate the potential of obtaining relative position updates with such a system by cross-correlating the measured to previously mapped gravity gradients over a known location. Both gravity gradient and gravity anomaly map matching have been analyzed to compare their relative merit. To restrict the scope of the problem, the analysis has been conducted with one element of the gravity gradient tensor in a one-dimensional cross correlation process. The accuracy of relative position updates are presented for various vehicle speeds and for a range of geodetic instrumentation performance levels.

Contents

A vehicle equipped with an integrated geodetic measurement system (inertial navigator, gravimeter, and gravity gradiometer) is assumed to travel a straight course. The following geodetic parameters are continuously mapped and recorded as a function of indicated position: either $T_Z = \partial T/\partial Z$ gravity anomaly, or $T_{XZ} = \partial^2 T/\partial X \partial Z$ gravity gradient element, where T is the anomalous gravity field potential. T_Z and T_{XZ} are the spatial derivatives of the potential as indicated above. The vehicle is assumed to travel along the horizontal (X) direction while mapping T_{XZ} and T_Z. The geodetic anomalies measured on a second pass are correlated with those of the first pass. The along track position error is assumed to be given by the distance shift at which the cross correlation peaks. The major assumptions are: 1) instrumentation noise consists of additive white noise on both passes; 2) the statistical characterization of the gravity field anomalies discussed in the next section is valid; and 3) only an along-track position error has been assumed. The cross-track error has been assumed small. In the actual application of gravity gradient matching techniques, the vehicle would follow a closed path to correct the position error about the two navigation coordinates.

Several different models for characterizing gravity and deflection anomalies have been presented in the literature.[1,2] The model used in this study is shown in Fig. 1 along with asymptotic power spectral densities associated with T_{XZ} and T_Z. In Fig. 1 the anomalous potential $T = W - U$; W = gravity potential; U = normal gravity potential (that of an equipotential elipsoid); The velocity V is in the X direction; $D_1 = 20$ naut. miles correlation distance assumed for predominant gravity anomaly breakpoint and $D_2 = 2.5$ naut. miles correlation distance assumed for predominant gradient anomaly breakpoint. The rms level of gravity anomaly T_Z is 40 milligals and that of T_{XZ} is 30 Eötvös units. Figures 2 and 3 show the steady-state relative positioning performance of a range of instrument accuracies over a range of vehicle velocities for the gravimeter and gradiometer systems respectively.

The geodetic anomalies sensed on the two passes are cross correlated based on position as determined by the navigation system. An along track velocity error stretches or compresses one record relative to the other. If the velocity error changes with time, so will the distortion of the position base thus adversely affecting the cross-correlation results.

The results presented assume zero velocity error, however, with a more sophisticated cross-correlation algorithm, this assumption can be relaxed for the gradiometer system. A system which utilizes a gradiometer to update the inertial navigation system for deflection of the vertical in real time inherently has well-behaved navigation errors[3] (especially velocity). Error buildup in the Schuler loop due to gravity anomalies are thus held to a very low level. If the inertial components (gyros and accelerometers) are of good quality, the resulting inertial system velocity errors will be well behaved, i.e., the Schuler error existing at the start of a matching run will remain relatively pure (unchanged as to am-

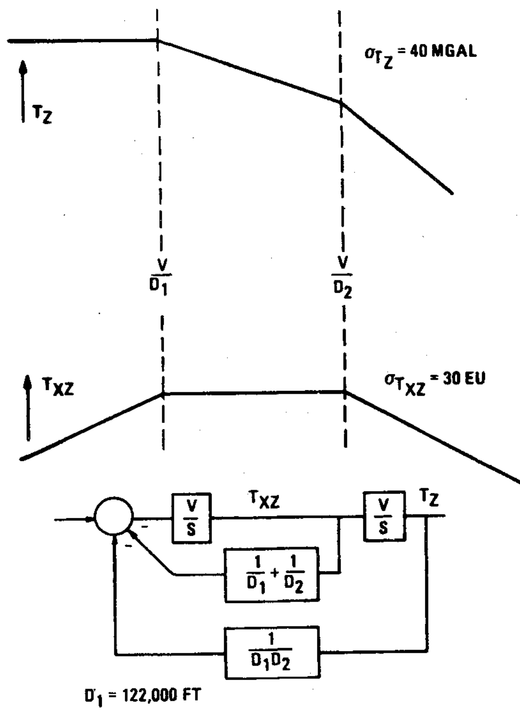

Fig. 1 Characterization of gradient and gravity anomalies.

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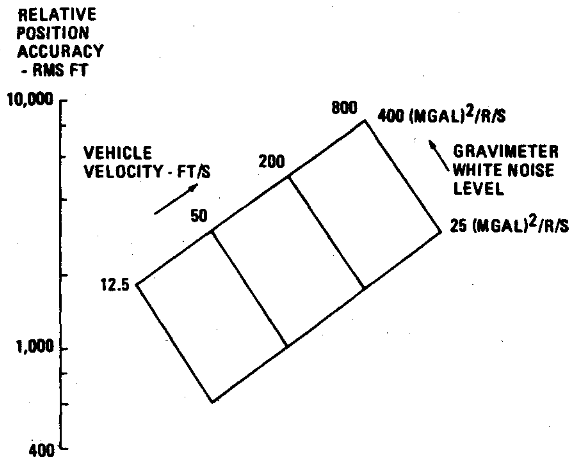

Fig. 2 Gravimeter system steady state relative position accuracy.

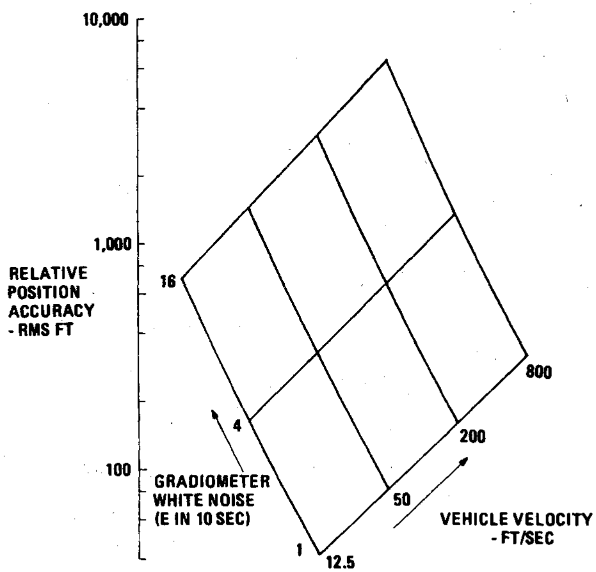

Fig. 3 Gradiometer system steady state relative position accuracy.

plitude and phase) during the run. The gravity gradient matching scheme requires a record length of 15,000 ft in order to approach the steady-state level of performance. Even at low ship speeds this distance is traveled in about 17 min (less than ¼ Schuler period). A cross-correlation algorithm could be designed for such a vehicle by assuming the velocity error has the form

$$V_e = V_0 + V_s \sin W_s t + V_c \cos W_s t \qquad (1)$$

where $V_0 = dc$ or low frequency component of velocity error; $V_s, V_c =$ coefficients of sine and cosine of Schuler components of error and $W_s =$ Schuler frequency. Setting $t = 0$ at the beginning of the matching run, the integral of Eq. (1) defines the cumulative position data base distortion.

$$P_e = V_0 t - V_s / W_s \cos W_s t + V_c W_s \sin W_s t = P_0 \qquad (2)$$

In this equation we assume that on the mapping pass, navigation control maintains negligible velocity error. An alternative point of view is that Eq. (2) characterizes the velocity error on the second pass relative to that on the first pass. The standard procedure for finding along-track position error is to cross correlate the geodetic parameters (gravity or gravity gradient anomalies(recorded as a function of position on the two passes. The value of along track position offset which maximizes the normalized cross correlation is the along track position error. By contrast if we consider Eq. (2), what we desire is not a one but a four-dimensional search to maximize the normalized cross correlation with respect to the parameters (P_0, V_0, V_s, V_c). The end result is that we not only solve for along track position error, but along track Schuler and quasi-*dc* velocity error relative to the first pass as well. To accommodate varying vehicle velocity, time relative to some start point on both passes must also be recorded along with position and appropriate geodetic parameters.

At the higher vehicle velocities, say 400 fps, the 15,000 ft record length is covered in 38 sec; as a result, the one-dimensional search algorithm would be appropriate. Several such fixes separated in time would be required to reset the Schuler or lower frequency components of velocity error. In general, the design of an appropriate cross correlation algorithm depends on the application, and especially on vehicle velocity.

Gravity anomaly matching has the following shortcomings compared to using gravity gradients: 1) the long record lengths required (>250,000 ft) and corresponding times ranging from hours for a slow moving vehicle to about 5 min for an 800 fps vehicle would, except at the higher speeds, present problems in maintaining a data base for the cross-correlation process; 2) without a gradiometer, horizontal anomalies would in general not be available in real time, consequently, the Schuler error present at the start of a matching run would not remain pure, but would change in amplitude and phase during the cross-correlation process thus complicating the data base modeling process.

References

[1] Shaw, L., Paul, I., and Henderson, P., *Journal of Geophysics Research,* Vol. 74, Aug. 1969, pp. 4259, 4265.

[2] Jordan, S. K., *IEEE Transactions on Aerospace and Electronic Systems,* Vol. AES-9, Sept. 1973, pp. 741, 752.

[3] Metzger, E. H. and Jircitano, A., "Analysis of Real Time Mapping of Horizontal and Vertical Gravity Anomalies Aboard a Moving Vehicle Such as an Aircraft," *International Symposium on Applications of Marine Geodesy*, Columbus, Ohio, June 1974.